IIT-JAM

Mathematics (MA)

2021

SECTION – A

MULTIPLE CHOICE QUESTIONS (MCQ)

Q. 1 – Q. 10 carry one mark each.

(1.) Let $0 < \alpha < 1$ be a real number. The number of differentiable functions $y:[0,1]\to[0,\infty)$, having continuous derivative on $[0,1]$ and satisfying

$y'(t) = (y(t))^{\alpha}, t \in [0,1]$,

$y(0) = 0$, is

(a.) Exactly one

(b.) Exactly two

(c.) Finite but more than two

(d.) Infinite

(2.) Let $P:\mathbb{R}\to\mathbb{R}$ be a continuous function such that $P(x) > 0$ for all $x \in \mathbb{R}$. Let y be a twice differentiable function on $\mathbb{R}$ satisfying $y''(x) + P(x)y'(x) - y(x) = 0$ for all $x \in \mathbb{R}$. Suppose that there exist two real numbers $a, b\,(a < b)$ such that $y(a) = y(b) = 0$. Then

(a.) $y(x) = 0$ for all $x \in [a,b]$

(b.) $y(x) > 0$ for all $x \in (a,b)$

(c.) $y(x) < 0$ for all $x \in (a,b)$

(d.) $y(x)$ changes sign on (a,b)

(3.) Let $f:\mathbb{R}\to\mathbb{R}$ be a continuous function satisfying $f(x) = f(x+1)$ for all $x \in \mathbb{R}$. Then

(a.) f is not necessarily bounded above

(b.) There exists a unique $x_0 \in \mathbb{R}$ such that $f(x_0 + \pi) = f(x_0)$

(c.) There is no $x_0 \in \mathbb{R}$ such that $f(x_0 + \pi) = f(x_0)$

(d.) There exist infinitely many $x_0 \in \mathbb{R}$ such that $f(x_0 + \pi) = f(x_0)$

(4.) Let $f:\mathbb{R}\to\mathbb{R}$ be a continuous function such that for all $x\in\mathbb{R}$,

$$\int_0^1 f(xt)\,dt = 0 \qquad (*)$$

Then

(a.) f must be identically 0 on the whole of $\mathbb{R}$

(b.) There is an f satisfying $(*)$ that is identically 0 on $(0,1)$ but not identically 0 on the whole of $\mathbb{R}$

(c.) There is an f satisfying $(*)$ that takes both positive and negative values

(d.) There is an f satisfying $(*)$ that is 0 at infinitely many points, but is not identically zero

(5.) Let p and t be positive real numbers. Let D_t be the closed disc of radius t centered at $(0,0)$, i.e., $D_t=\{(x,y)\in\mathbb{R}^2 : x^2+y^2\le t^2\}$. Define $I(p,t)=\iint_{D_t}\frac{dxdy}{(p^2+x^2+y^2)^p}$.

Then $\lim\limits_{t\to\infty} I(p,t)$ is finite

(a.) Only if $p>1$

(b.) Only if $p=1$

(c.) Only if $p<1$

(d.) For no value of p

(6.) How many elements of the group $\mathbb{Z}_{50}$ have order 10?

(a.) 10

(b.) 4

(c.) 5

(d.) 8

(7.) For every $n\in\mathbb{N}$, let $f_n:\mathbb{R}\to\mathbb{R}$ be a function. From the given choices, pick the statement that is the negation of "For every $x\in\mathbb{R}$ and for every real number $\varepsilon>0$, there exists an integer $N>0$ such that $\sum\limits_{i=1}^{p}\left|f_{N+i}(x)\right|<\varepsilon$ for every integer $p>0$."

(a.) For every $x\in\mathbb{R}$ and for every real number $\varepsilon>0$, there does not exists any integer $N>0$ such that $\sum_{i=1}^{p}\left|f_{N+i}(x)\right|<\varepsilon$ for every integer $p>0$.

(b.) For every $x\in\mathbb{R}$ and for every real number $\varepsilon>0$, there exists an integer $N>0$ such that $\sum_{i=1}^{p}\left|f_{N+i}(x)\right|\ge\varepsilon$ for some integer $p>0$.

(c.) There exists $x\in\mathbb{R}$ and there exists a real number $\varepsilon>0$ such that for every integer $N>0$, there exists an integer $p>0$ for which the inequality $\sum_{i=1}^{p}\left|f_{N+i}(x)\right|\ge\varepsilon$ holds (d.) There exists $x \in \mathbb{R}$ and there exists a real number $\varepsilon > 0$ such that for every integer $N > 0$ and for every integer $p > 0$ the inequality $\sum_{i=1}^{p} |f_{N+i}(x)| \geq \varepsilon$ holds

(8.) Which one of the following subsets of $\mathbb{R}$ has a non-empty interior?

(a.) The set of all irrational number in $\mathbb{R}$

(b.) The set $\{a \in \mathbb{R} : \sin(a) = 1\}$

(c.) The set $\{b \in \mathbb{R} : x^2 + bx + 1 = 0$ has distinct roots $\}$

(d.) The set of all rational numbers in $\mathbb{R}$

(9.) For an integer $k \geq 0$, let P_k denote the vector space of all real polynomials in one variable of degree less than or equal to k. Define a linear transformation $T : P_2 \to P_3$ by $Tf(x) = f''(x) + xf(x)$.

Which one of the following polynomials is not in the range of T?

(a.) $x + x^2$

(b.) $x^2 + x^3 + 2$

(c.) $x + x^3 + 2$

(d.) $x + 1$

(10.) Let $n > 1$ be an integer. Consider the following two statements for an arbitrary $n \times n$ matrix A with complex entries.

I. If $A^k = I_n$ for some integer $k \geq 1$, then all the eigenvalues of A are k^{th} roots of unity.

II. If, for some integer $k \geq 1$, all the eigenvalues of A are k^{th} roots of unity, then $A^k = I_n$.

Then

(a.) Both I and II are TRUE

(b.) I is TRUE but II is FALSE

(c.) I is FALSE but II is TRUE

(d.) Neither I nor II is TRUE

Q.11 – Q.30 carry two marks each.

(11.) Let $M_n(\mathbb{R})$ be the real vector space of all $n \times n$ matrices with real entries, $n \geq 2$. Let $A \in M_n(\mathbb{R})$. Consider the subspace W of $M_n(\mathbb{R})$ spanned by $\{I_n, A, A^2, \ldots\}$. Then the dimension of W over $\mathbb{R}$ is necessarily

(a.) ∞

(b.) n^2

(c.) n

(d.) At most n

(12.) Let y be the solution of

$$(1+x)y''(x)+y'(x)-\frac{1}{1+x}y(x)=0\ ,\ \ x\in(-1,\infty)\ ,\ \ y(0)=1, y'(0)=0\ .$$

Then

(a.) y is bounded on $(0,\infty)$

(b.) y is bounded on $(-1,0]$

(c.) $y(x)\geq 2$ on $(-1,\infty)$

(d.) y attains its minimum at $x=0$

(13.) Consider the surface

$S=\{(x,y,xy)\in\mathbb{R}^3 : x^2+y^2\leq 1\}$.

Let $\vec{F}=y\hat{i}+x\hat{j}+\hat{k}$.

If $\hat{n}$ is the continuous unit normal field to the surface S with positive z-component, then $\iint_S \vec{F}\cdot\hat{n}\,dS$ equals

(a.) $\frac{\pi}{4}$

(b.) $\frac{\pi}{2}$

(c.) π

(d.) 2π

(14.) Consider the following statements.

I. The group $(\mathbb{Q},+)$ has no proper subgroup of finite index.

II. The group $(\mathbb{C}\setminus\{0\},\cdot)$ has no proper subgroup of finite index.

Which one of the following statements is true?

(a.) Both I and II are TRUE

(b.) I is TRUE but II is FALSE

(c.) II is TRUE but I is FALSE

(d.) Neither I nor II is TRUE

(15.) Let $f:\mathbb{N}\to\mathbb{N}$ be a bijective map such that $\sum_{n=1}^{\infty}\frac{f(n)}{n^2}<+\infty$. The number of such bijective map is

(a.) Exactly one

(b.) Zero

(c.) Finite but more than one

(d.) Infinite

(16.) Define $S = \lim_{n\to\infty}\left(1-\frac{1}{2^2}\right)\left(1-\frac{1}{3^2}\right)....\left(1-\frac{1}{n^2}\right)$.

Then

(a.) $S = \frac{1}{2}$

(b.) $S = \frac{1}{4}$

(c.) $S = 1$

(d.) $S = \frac{3}{4}$

(17.) Let $f:\mathbb{R}\to\mathbb{R}$ be an infinitely differentiable function such that for all $a,b\in\mathbb{R}$ with $a<b$, $\frac{f(b)-f(a)}{b-a} = f'\left(\frac{a+b}{2}\right)$.

Then

(a.) f must be a polynomial of degree less than or equal to 2

(b.) f must be a polynomial of degree greater than 2

(c.) f is not a polynomial

(d.) f must be a linear polynomial

(18.) Consider the function

$$f(x) = \begin{cases} 1 & \text{if } x\in(\mathbb{R}\setminus\mathbb{Q})\cup\{0\}, \\ 1-\frac{1}{p} & \text{if } x=\frac{n}{p}, n\in\mathbb{Z}\setminus\{0\}, \\ & p\in\mathbb{N} \text{ and } \gcd(n,p)=1 \end{cases}$$

Then

(a.) All $x\in\mathbb{Q}\setminus\{0\}$ are strict local minima for f

(b.) f is continuous at all $x\in\mathbb{Q}$

(c.) f is not continuous at all $x\in\mathbb{R}\setminus\mathbb{Q}$

(d.) f is not continuous at $x=0$

(19.) Consider the family of curves $x^2-y^2=ky$ with parameter $k\in\mathbb{R}$. The equation of the orthogonal trajectory to this family passing through $(1,1)$ is given by

(a.) $x^3+3xy^2=4$

(b.) $x^2+2xy=3$

(c.) $y^2+2x^2y=3$

(d.) $x^3 + 2xy^2 = 3$

(20.) Which one of the following statements is true?

(a.) Exactly half of the elements in any even order subgroup of S_5 must be even permutations

(b.) Any abelian subgroup of S_5 is trivial

(c.) There exists a cyclic subgroup of S_5 of order 6

(d.) There exists a normal subgroup of S_5 of index 7

(21.) Let $f:[0,1]\to[0,\infty)$ be a continuous function such that $\left(f(t)\right)^2 < 1+2\int_0^t f(s)\,ds$, for all $t\in[0,1]$.

Then

(a.) $f(t) < 1+t$ for all $t\in[0,1]$

(b.) $f(t) > 1+t$ for all $t\in[0,1]$

(c.) $f(t) = 1+t$ for all $t\in[0,1]$

(d.) $f(t) < 1+\frac{t}{2}$ for all $t\in[0,1]$

(22.) Let A be an $n\times n$ invertible matrix and C be an $n\times n$ nilpotent matrix. If $X=\begin{pmatrix} X_{11} & X_{12} \\ X_{21} & X_{22} \end{pmatrix}$ is a $2n\times 2n$ matrix (each X_{ij} being $n\times n$) that commutes with the $2n\times 2n$ matrix $B=\begin{pmatrix} A & 0 \\ 0 & C \end{pmatrix}$, then

(a.) X_{11} and X_{22} are necessarily zero matrices

(b.) X_{12} and X_{21} are necessarily zero matrices

(c.) X_{11} and X_{21} are necessarily zero matrices

(d.) X_{12} and X_{22} are necessarily zero matrices

(23.) Let $D\subseteq\mathbb{R}^2$ be defined by $D=\mathbb{R}^2\setminus\{(x,0):x\in\mathbb{R}\}$.

Consider the function $f:D\to\mathbb{R}$ defined by $f(x,y)=x\sin\frac{1}{y}$.

Then

(a.) f is a discontinuous function on D

(b.) f is a continuous function on D and cannot be extended continuously to any point outside D

(c.) f is a continuous function on D and can be extended continuously to $D\cup\{(0,0)\}$

(d.) f is a continuous function on D and can be extended continuously to the whole of $\mathbb{R}^2$

(24.) Which one of the following statements is true?

(a.) $(\mathbb{Z},+)$ is isomorphic to $(\mathbb{R},+)$

(b.) $(\mathbb{Z},+)$ is isomorphic to $(\mathbb{Q},+)$

(c.) $(\mathbb{Q}/\mathbb{Z},+)$ is isomorphic to $(\mathbb{Q}/2\mathbb{Z},+)$

(d.) $(\mathbb{Q}/\mathbb{Z},+)$ is isomorphic to $(\mathbb{Q},+)$

(25.) Let y be a twice differentiable function on $\mathbb{R}$ satisfying

$y''(x) = 2 + e^{-|x|}, x \in \mathbb{R}$,

$y(0) = -1, y'(0) = 0$.

Then,

(a.) $y = 0$ has exactly one root

(b.) $y = 0$ has exactly two roots

(c.) $y = 0$ has more than two roots

(d.) There exists an $x_0 \in \mathbb{R}$ such that $y(x_0) \geq y(x)$ for all $x \in \mathbb{R}$

(26.) Let $f:[0,1] \to [0,1]$ be a non-constant continuous function such that $f \circ f = f$. Define $E_f = \{x \in [0,1] : f(x) = x\}$. Then

(a.) E_f is neither open nor closed

(b.) E_f is an interval

(c.) E_f is empty

(d.) E_f need not be an interval

(27.) Let g be an element of S_7 such that g commutes with the element $(2,6,4,3)$. The number of such g is

(a.) 6

(b.) 4

(c.) 24

(d.) 48

(28.) Let G be a finite abelian group of odd order.

Consider the following two statements:

I. The map $f: G \to G$ defined by $f(g) = g^2$ is a group isomorphism.

II. The product $\prod_{g \in G} g = e$

(a.) Both I and II are TRUE

(b.) I is TRUE but II is FALSE

(c.) II is TRUE but I is FALSE

(d.) Neither I nor II is TRUE

(29.) Let $n \geq 2$ be an integer. Let $A : \mathbb{C}^n \to \mathbb{C}^n$ be the linear transformation defined by $A(z_1, z_2, \ldots, z_n) = (z_n, z_1, z_2, \ldots, z_{n-1})$.

Which one of the following statements is true for every $n \geq 2$?

(a.) A is nilpotent

(b.) All eigenvalues of A are of modulus 1

(c.) Every eigenvalue of A is either 0 or 1

(d.) A is singular

(30.) Consider the two series

I. $\sum_{n=1}^{\infty} \frac{1}{n^{1+(1/n)}}$ and

II. $\sum_{n=1}^{\infty} \frac{1}{n^{2-n^{1/n}}}$

Which one of the following holds?

(a.) Both I and II converges

(b.) Both I and II diverge

(c.) I converges and II diverges

(d.) I diverges and II converges

SECTION – B

MULTIPLE SELECT QUESTIONS (MSQ)

Q. 31 – Q. 40 carry two marks each.

(31.) Let $f : \mathbb{R} \to \mathbb{R}$ be a function with the property that for every $y \in \mathbb{R}$, the value of the expression $\sup_{x \in \mathbb{R}} [xy - f(x)]$ is finite. Define $g(y) = \sup_{x \in \mathbb{R}} [xy - f(x)]$ for $y \in \mathbb{R}$. Then

(a.) g is even if f is even

(b.) f must satisfy $\lim_{|x| \to \infty} \frac{f(x)}{|x|} = +\infty$

(c.) g is odd if f even

(d.) f must satisfy $\lim_{|x| \to \infty} \frac{f(x)}{|x|} = -\infty$

(32.) Consider the equation

$x^{2021} + x^{2020} + \ldots + x - 1 = 0$.

Then

(a.) All real roots is positive

(b.) Exactly one real root is positive

(c.) Exactly one real root is negative

(d.) No real root is positive

(33.) Let $D=\mathbb{R}^2 \setminus \{(0,0)\}$. Consider the two function $u,v: D\to\mathbb{R}$ defined by $u(x,y)=x^2-y^2$ and $v(x,y)=xy$. Consider the gradients ∇u and ∇v of the functions u and v, respectively. Then

(a.) ∇u and ∇v are parallel at each point (x,y) of D

(b.) ∇u and ∇v are perpendicular at each point (x,y) of D

(c.) ∇u and ∇v do not exist at some points (x,y) of D

(d.) ∇u and ∇v at each point (x,y) of D span $\mathbb{R}^2$

(34.) Consider the two functions $f(x,y)=x+y$ and $g(x,y)=xy-16$ defined on $\mathbb{R}^2$. Then

(a.) The function f has no global extreme value subject to the condition $g=0$

(b.) The function f attains global extreme values at $(4,4)$ and $(-4,-4)$ subject to the condition $g=0$

(c.) The function g has no global extreme value subject to the condition $f=0$

(d.) The function g has a global extreme value at $(0,0)$ subject to the condition $f=0$

(35.) Let $f:(a,b)\to\mathbb{R}$ be a differentiable function on (a,b).

Which of the following statements is/are true?

(a.) $f'>0$ in (a,b) implies that f is increasing in (a,b)

(b.) f is increasing in (a,b) implies that $f'>0$ in (a,b)

(c.) If $f'(x_0)>0$ for some $x_0\in(a,b)$, then there exists a $\delta>0$ such that $f(x)>f(x_0)$ for all $x\in(x_0,x_0+\delta)$

(d.) If $f'(x_0)>0$ for some $x_0\in(a,b)$, then f is increasing in a neighborhood of x_0

(36.) Let G be a finite group of order 28. Assume that G contains a subgroup of order 7.

Which of the following statements is/are true?

(a.) G contains a unique subgroup of order 7

(b.) G contains a normal subgroup of order 7

(c.) G contains no normal subgroup of order 7

(d.) G contains at least two subgroups of order 7

(37.) Which of the following subsets of $\mathbb{R}$ is/are connected?

(a.) The set $\{x \in \mathbb{R} : x \text{ is irrational}\}$

(b.) The set $\{x \in \mathbb{R} : x^3 - 1 \geq 0\}$

(c.) The set $\{x \in \mathbb{R} : x^3 + x + 1 \geq 0\}$

(d.) The set $\{x \in \mathbb{R} : x^3 - 2x + 1 \geq 0\}$

(38.) Consider the four functions from $\mathbb{R}$ to $\mathbb{R}$

$f_1(x) = x^4 + 3x^3 + 7x + 1$,

$f_2(x) = x^3 + 3x^2 + 4x$,

$f_3(x) = \arctan(x)$ and

$$f_4(x) = \begin{cases} x & \text{if } x \notin \mathbb{Z} \\ 0 & \text{if } x \in \mathbb{Z} \end{cases}$$

Which of the following subsets of $\mathbb{R}$ are open?

(a.) The range of f_1

(b.) The range of f_2

(c.) The range of f_3

(d.) The range of f_4

(39.) Let V be a finite dimensional vector space and $T : V \to V$ be a linear transformation. Let $\mathcal{R}(T)$ denote the range of T and $\mathcal{N}(T)$ denote the null space $\{v \in V : Tv = 0\}$ of T. If $rank(T) = rank(T^2)$, then which of the following is/are necessarily true?

(a.) $\mathcal{N}(T) = \mathcal{N}(T^2)$

(b.) $\mathcal{R}(T) = \mathcal{R}(T^2)$

(c.) $\mathcal{N}(T) \cap \mathcal{R}(T) = \{0\}$

(d.) $\mathcal{N}(T) = \{0\}$

(40.) Let $m > 1$ and $n > 1$ be integers. Let A be an $m \times n$ matrix such that for some $m \times 1$ matrix b_1, the equation $Ax = b_1$ has infinitely many solutions. Let b_2 denote an $m \times 1$ matrix different from b_1. Then $Ax = b_2$ has

(a.) Infinitely many solutions for some b_2

(b.) A unique solution for some b_2

(c.) No solution for some b_2

(d.) Finitely many solutions for some b_2

SECTION – C
NUMERICAL ANSWER TYPE (NAT)
Q. 41 – Q. 50 carry one mark each.

(41.) The number of cycles of length 4 in S_6 is__________.

(42.) The value of $\lim_{n\to\infty}\left(3^n+5^n+7^n\right)^{\frac{1}{n}}$ is___________

(43.) Let $B=\left\{(x,y,z)\in\mathbb{R}^3 : x^2+y^2+z^2\le 1\right\}$ and define $u(x,y,z)=\sin\left(\left(1-x^2-y^2-z^2\right)^2\right)$ for $(x,y,z)\in B$. Then the value of $\iiint_B\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)dx\,dy\,dz$ is ________.

(44.) Consider the subset

$S=\left\{(x,y): x^2+y^2>0\right\}$ of $\mathbb{R}^2$.

Let $P(x,y)=\frac{y}{x^2+y^2}$ and

$Q(x,y)=-\frac{x}{x^2+y^2}$ for $(x,y)\in S$.

If C denotes the unit circle traversed in the counter-clockwise direction, then the value of $\frac{1}{\pi}\int_C (Pdx+Qdy)$ is __________.

(45.) Consider the set $A=\{a\in\mathbb{R} : x^2=a(a+1)(a+2)$ has a real root $\}$. The number of connected components of A is ________.

(46.) Let V be the real vector space of all continuous functions $f:[0,2]\to\mathbb{R}$ such that the restriction of f to the interval $[0,1]$ is a polynomial of degree less than or equal to 2, the restriction of f to the interval $[1,2]$ is a polynomial of degree less than or equal to 3 and $f(0)=0$. Then the dimensions of V is equal to ————.

(47.) The number of group homomorphisms from the group $\mathbb{Z}_4$ to the group S_3 is ______.

(48.) Let $y:\left(\frac{9}{10},3\right)\to\mathbb{R}$ be a differentiable function satisfying $(x-2y)\frac{dy}{dx}+(2x+y)=0,\ x\in\left(\frac{9}{10},3\right)$, and $y(1)=1$. Then $y(2)$ equal____________.

(49.) Let $\vec{F}=(y+1)e^{y}\cos(x)\hat{i}+(y+2)e^{y}\sin(x)\hat{j}$ be a vector field in $\mathbb{R}^2$ and C be a continuously differentiable path with the starting point $(0,1)$ and the end point $\left(\frac{\pi}{2},0\right)$. Then $\int_C \vec{F}\cdot d\vec{r}$ equals ______.

(50.) The value of $\frac{\pi}{2}\lim_{n\to\infty}\cos\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{8}\right)\ldots\cos\left(\frac{\pi}{2^{n+1}}\right)$ is ________.

Q. 51 – Q. 60 carry two marks each.

(51.) The number of elements of order two in the group S_4 is equal to ________.

(52.) The least possible value of k, accurate up to two decimal places, for which the following problem

$$y''(t)+2y'(t)+ky(t)=0, t\in\mathbb{R},$$

$$y(0)=0, y(1)=0, y\left(\frac{1}{2}\right)=1,$$

has a solution is ______.

(53.) Consider those continuous function $f:\mathbb{R}\to\mathbb{R}$ that have the property that given any $x\in\mathbb{R}$, $f(x)\in\mathbb{Q}$ if and only if $f(x+1)\in\mathbb{R}\setminus\mathbb{Q}$. The number of such functions is______________.

(54.) The largest positive number a such that $\int_0^5 f(x)\,dx+\int_0^3 f^{-1}(x)\,dx\geq a$ for every strictly increasing surjective continuous function $f:[0,\infty)\to[0,\infty)$ is _______.

(55.) Define the sequence

$$s_n=\begin{cases}\dfrac{1}{2^n}\displaystyle\sum_{j=0}^{n-2}2^{2j} & \text{if } n>0 \text{ is even}\\ \dfrac{1}{2^n}\displaystyle\sum_{j=0}^{n-1}2^{2j} & \text{if } n>0 \text{ is odd}\end{cases}.$$

Define $\sigma_m=\frac{1}{m}\sum_{n=1}^{m}s_n$. The number of limit points of the sequence $\{\sigma_m\}$ is __________.

(56.) The determinant of the matrix $\begin{pmatrix}2021 & 2020 & 2020 & 2020\\ 2021 & 2021 & 2020 & 2020\\ 2021 & 2021 & 2021 & 2020\\ 2021 & 2021 & 2021 & 2021\end{pmatrix}$ is __________.

(57.) The value of $\lim_{n\to\infty}\int_0^1 e^{x^2}\sin(nx)\,dx$ is ______.

(58.) Let S be the surface defined by $\{(x,y,z)\in\mathbb{R}^3 : z=1-x^2-y^2, z\geq 0\}$.

Let $\vec{F}=-y\hat{i}+(x-1)\hat{j}+z^2\hat{k}$ and $\hat{n}$ be the continuous unit normal field to the surface S with positive z-component. Then the value of $\frac{1}{\pi}\iint_S(\nabla\times\vec{F})\cdot\hat{n}\,dS$ is__________.

(59.) Let $A=\begin{pmatrix}2 & -1 & 3\\ 2 & -1 & 3\\ 3 & 2 & -1\end{pmatrix}$.

Then the largest eigenvalue of A is ______.

(60.) Let $A=\begin{pmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & -1 & 0\\ 0 & 0 & 0 & -1\end{pmatrix}$. Consider the linear map T_A from the real vector space $M_4(\mathbb{R})$ to itself defined by $T_A(X)=AX-XA$, for all $X\in M_4(\mathbb{R})$. The dimension of the range of T_A is ________.